**MATHEMATICS SPECIALIST**

**MAWA Year 12 Examination 2019**

**Calculator-free**

# Marking Key

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The release date for this exam and marking scheme is 14th June.

**Question 1(a) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| If    then    Alternatively we note that with and .  Then  as before | |
| Mathematical behaviours | Marks |
| * calculates  correctly * calculates correctly   (1 mark for showing real part zero and 1 mark for correct value of imaginary part) | 1  1+1 |

**Question 1(b) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| Since    then  which is real and negative. Hence | |
| Mathematical behaviours | Marks |
| * calculates  correctly | 1 |

**Question 2 (a) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| Augmented matrix = | |
| Mathematical behaviours | Marks |
| * correctly transfers coefficients of equations to augmented matrix | 1 |

**Question 2 (b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| After and the system is reduced to:      After the system is further reduced to | |
| Mathematical behaviours | Marks |
| * correctly reduces *x*-components to 0 for rows 2 and 3 (or equivalent) * correctly reduces *y*-component to 0 for row 3 (or equivalent) | 2  1 |

**Question 2 (c) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| From the augmented matrix form we deduce that  (i) for unique solution,  (ii) for no solution and  (iii) for infinitely many solutionsand | |
| Mathematical behaviours | Marks |
| * correctly determines value of  for a unique solution * correctly determines values of  and  that means there is no solution * correctly states the values of  and  for infinitely many solutions | 1  1  1 |

**Question 2 (d) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| When and the augmented matrix becomes    Then the second equation gives  and first equation then leads to  Hence the general solution of the equations is | |
| Mathematical behaviours | Marks |
| * determines in terms of (or vice-versa) * determines the value of * states the general solution in terms of a suitable parameter | 1  1  1 |

**Question 3 (5 marks)**

|  |  |
| --- | --- |
| Solution | |
| If then so that  For this to be the same as the linear function then comparison of the coefficients of and the constant requires that  and  Hence  so . If  then  so  If  then  is arbitrary  We conclude that either or with any real number | |
| Mathematical behaviours | Marks |
| * derives equation for the inverse * compares coefficients to determine the equations for and * solves for * derives correct solution for * dervies correct solution for | 1  1  1  1  1 |

**Question 4 (4 marks)**

|  |  |
| --- | --- |
| Solution | |
| Let in which case ; additionally  Now    as required | |
| Mathematical behaviours | Marks |
| * writes down an appropriate form for and hence * derives an expression for * in quotient multiplies through by the complex conjugate * draws a valid conclusion | 1  1  1  1 |

**Question 5 (a) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| is defined if  If then  if  If then  if  So is defined for | |
| Mathematical behaviours | Marks |
| * obtains positivity requirement for * obtains lower and upper limits of the domain | 1  1+1 |

**Question 5 (b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * displays general shape of the graph * indicates maximum at * makes clear the non-differentiability at the maximum point | 1  1  1 |

**Question 6 (a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| print grid 3d paper free2 | |
| Mathematical behaviours | Marks |
| * correctly sketches the triangle on the plane with all three intercepts cutting the axes at  (-1 for one mistake) | 2 |

**Question 6 (b) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| Substituting gives:  Now LHS so the given point lies on the plane | |
| Mathematical behaviours | Marks |
| * correctly substitutes point into equation and confirms value | 1 |

**Question 6 (c) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| The vector **q** = 4**i****j****k** is perpendicular to *Q*.  Since **v** **q**, it can be concluded that **v** is parallel to **q**  As such **v** must also be perpendicular to *Q*. | |
| Mathematical behaviours | Marks |
| * recognises that **v** is a scalar multiple of **w** * concludes that **v** is parallel to **w** and so must also be perpendicular to *Q* | 1  1 |

**Question 6 (d) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Equation for so that | |
| Mathematical behaviours | Marks |
| * writes down equation with same coefficients * shows how to incorporate the fact that the required plane includes the given point | 1  1 |

**Question 6 (e) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| We can find **w** by forming the vector product    This vector, or any non-zero multiple of it, is the required perpendicular vector. | |
| Mathematical behaviours | Marks |
| * makes clear the need to construct a vector product * computes the vector product correctly | 1  1 |

**Question 7 (6 marks)**

|  |  |
| --- | --- |
| Solution | |
| First note that    Then  for  by de Moivre’s theorem  Hence the five roots are  where  Restricting the argument to the stated domain leaves | |
| Mathematical behaviours | Marks |
| * writes in a suitable polar form (1 for modulus, 1 for argument) * uses de Moivre’s theorem appropriately * writes down the five required roots (-1 for one mistake) * calculates all the arguments so that they lie in the appropriate given range | 1+1  1  2  1 |

**Question 8 (6 marks)**

|  |  |
| --- | --- |
| Solution | |
| The graph of is obtained by shifting the graph of 3 units to the right.  The graph of is obtained by shifting the graph of 3 units down.  The graph of is the same as the graph of for and then that part is reflected across the y-axis. | |
| Mathematical behaviours | Marks |
| * displays the correct geometric transformations * plots the graphs reasonably accurately | 1+1+1  1+1+1 |